Office Hours, Seat Numbers

Thursday, September 23, 2021 12:16 PM

Mayer Hall 3210

Revelle Plaza Self-Study Tent Thursdays 11-12

Seat M-24

K-23

Temperature, Heat

Thursday, September 23, 2021 12:33 PM

- Thermal energy is the total energy of its atoms and molecules
- More mass at the same temperature has more thermal energy

Temperature

Tuesday, September 28, 2021 12:44 PM

- Symbol : T
- Affects thermal energy of the object
- Temperature measures how fast the atoms are moving
  - how much energy does each atom have?
- Celsius or Kelvin units
  - 0 Kelvin is lowest temperature (absolute 0), atoms stop moving
  - $T_K = T_c + 273$
- No matter the unit, temperature always measures the energy of atoms.

Heat, Thermal Energy

Tuesday, September 28, 2021 12:52 PM

- Heat energy ≠ temperature
  - an Ice cold lake has more thermal energy than you
  - jumping in, thermal energy flows from you to the lake
  - Heat (Q) is the flow of thermal energy
  - Heat flows from high temperature to low temperature
- Matter contains thermal energy
  - measured in joules
  - heat in (+), heat out (-)

Thermodynamics, 0th Law

Tuesday, September 28, 2021 12:57 PM

- Study of heat transfer between objects
- 0th Law:
  - If bodies A, B are in thermal equilibrium with a third body T, then A, B are in thermal equilibrium with each other.
  - Heat flow: High T → Low T

Specific Heat (Temperature Change)

Tuesday, September 28, 2021 1:02 PM

Heat capacity: C is how fast an object changes temperature:

$$Q = C\Delta T = C(T_f - T_i)$$

specifically: $Q = mc\Delta T = mc(T_f - T_i)$

where c is the object's specific heat.

and Q is the heat flow

Latent Heat (Phase Change)

Tuesday, September 28, 2021 1:09 PM

- To change the phase of an object, heat required:

$$Q = mL$$

where L is the amount of energy per mass to complete the phase change, Latent Heat

- Liquid ⟷ Gas: L_V heat of vaporization

  Solid ⟷ Liquid: L_F heat of fusion.

Total Internal Thermal Energy

Tuesday, September 28, 2021 1:09 PM

Internal Thermal Energy: $E_{int} = mcT$ where T is the absolute temperature in kelvin

Solving a Thermal Energy System

Thursday, September 30, 2021 12:22 PM

- Total thermal energy of a system must stay the same:

Given a thermal system with objects $\{1 \ldots k\}$:

$$\sum_{n=1}^{k} Q_n = 0$$

for some known T_i and unknown T_f, solve for T_f

Ex:

Horseshoe	1.5kg	0.447	650°C
Water	6kg	4.337	35°C

$$Q_H = 1.5 \cdot 0.447 \cdot (T_f - 650) \qquad Q_w = 6 \cdot 4.337 \cdot (T_f - 35)$$

$$Q_H + Q_w = 0 \rightarrow 1.5 \cdot 0.447 (T_f - 650) + 6 \cdot 4.337 \cdot (T_f - 35) = 0$$

solve for T_f

Thermal Expansion (Linear, Volume)

Tuesday, September 28, 2021 1:39 PM

- When objects increase in thermal energy, the exibit thermal expansion.
- For a metal rod of length L:

  $\Delta L = \alpha L \Delta T$

  where α is the coefficient of linear expansion

- For some solid or liquid volume:

  $\Delta V = \beta V \Delta T$

  where β is the coefficient of volume expansion

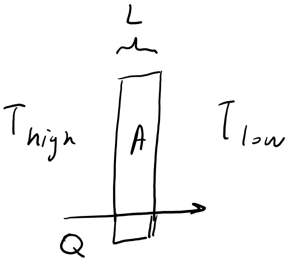
  generally $\beta = 3\alpha$ for some material

Methods of Heat Transfer (Conduction, Convection, Radiation)

Thursday, September 30, 2021 12:45 PM

- Conduction:
  - transfer by contact

$$\text{Watts}\left\{\frac{\overset{\text{Joules}}{\Delta Q}}{\underset{\text{seconds}}{\Delta t}}\right. = kA\,\frac{\Delta T}{L}$$



Q - heat flow
t - time
A - contact area
T - temperature difference
L - thickness

$$R = \frac{L}{k}\ ;\quad \text{Thermal Resistance}$$

- Convection:
  - transfer by fluid motion
- Radiation:
  - transfr by electromagnetic waves
  - Emissivity, ε, measures how good of a radiator

$$\frac{\Delta Q}{\Delta t} = \varepsilon\sigma A\left(T_{obj}{}^4 - T_{env}{}^4\right)$$

Q - heat flow
t - time
ε - emissivity
σ - constant
A - area
T_{obj} - temp object in K
T_{env} - temp env in K

Thermal Work

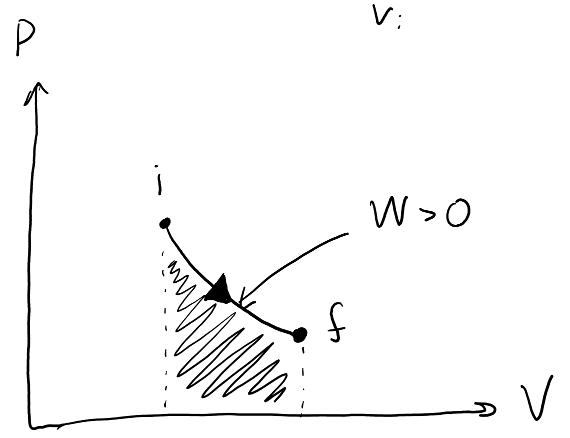
Thursday, September 30, 2021 1:12 PM

- With fluids:

$$P = \frac{F}{A}, \quad W = \int pA\,dx = \int p\,dV$$

- so the work with a fluid changing volume:

$$W = \int \partial W = \int_{V_i}^{V_f} p\,\partial V$$



- if the volume does not change, there is no work

1st Law of Thermodynamics

Thursday, September 30, 2021 1:21 PM

- 1st Law: Conservation of Energy
  - Generally:

$$\partial E_{int} = \partial Q - \partial W$$

$$\Delta E_{int} = Q - W$$

  - Adiabatic process: no transfer of thermal energy (ex. can of compersed air)

$$Q = 0, \quad \Delta E_{int} = -W$$

  - Free expansion: no heat transfer, not volume expansion

$$Q = W = 0, \quad \Delta E_{int} = 0$$

  - Constant Volume process: volume held constant

$$W = 0, \quad \Delta E_{int} = Q$$

  - Cyclical process: system loops back to initial state

$$Q = W \neq 0, \quad \Delta E_{int} = 0$$

  - Isothermal process:

$$\Delta E_{int} = 0, \quad Q = W$$

Properties of Ideal Gasses

Tuesday, October 5, 2021 1:24 PM

Ideal Gas Law

Thursday, September 30, 2021 1:39 PM

- Assume all gasses behave like ideal gasses
- Ideal Gas Law:

$PV = nrT$

P - pressure
V - volume
n - mols
r - gas constant
t - temperature

$PV = Nk_B T$ where N - number of molecules, k_B is constant (Boltzmann)

Work Done by Ideal Gas, Isothermal System

Tuesday, October 5, 2021 12:46 PM

- For Ideal Gasses:

$$W = \int_{V_i}^{V_f} p \, dV \quad \& \quad p = \frac{nrT}{V}$$

thus:

$$W = nr \int_{V_i}^{V_f} \frac{T}{V} \, dV$$

- If temperature is constant (Isothermal):

$$W = nrT \int_{V_i}^{V_f} \frac{1}{V} \, dV = nrT \ln(V) \Big|_{V_i}^{V_f} = nrT \ln\left(\frac{V_f}{V_i}\right)$$

Summary of Work Done By Ideal Gasses

Tuesday, October 5, 2021 12:49 PM

- Constant volume: $W = p\Delta V = 0$
- Constant pressure: $W = \int_{V_i}^{V_f} p\,dV = p\Delta V$
- Constant temperature: $W = nrT\ln\left(\frac{V_f}{V_i}\right)$
- Generally:

$$W = \int_{V_i}^{V_f} p\,dV = nr\int_{V_i}^{V_f} \frac{T}{V}\,dV$$

RMS Speed

Tuesday, October 5, 2021 1:03 PM

- Given an ideal gas constrained in volume V at temperature T and with n particles of molar mass M and average velocity v_{avg}

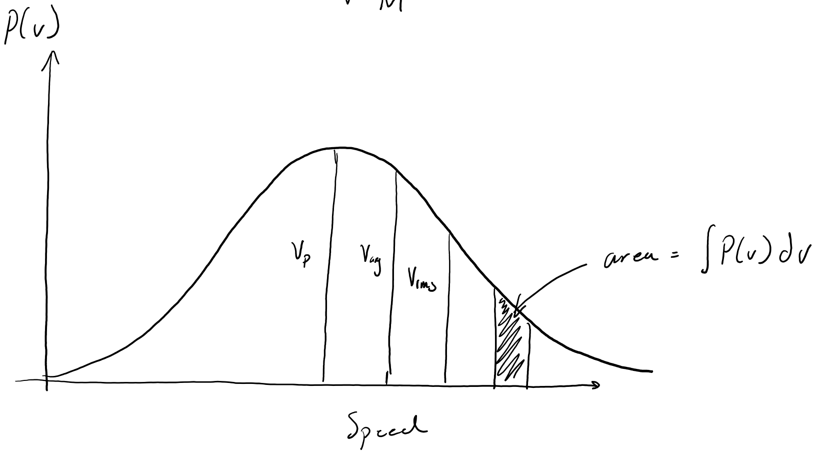
$$p = \left(\frac{Mn}{3V}\right)(v^2)_{avg} \quad \text{and since} \quad p = \frac{nrT}{V} \text{, then}$$

$$\frac{nrT}{V} = \left(\frac{Mn}{3V}\right)(v^2)_{avg} \rightarrow (v^2)_{avg} = \frac{3rT}{M}$$

where M is molar mass
n is number of moles

- RMS velocity: $V_{rms} = \sqrt{(v^2)_{avg}}$

thus: $V_{rms} = \sqrt{\frac{3rT}{M}}$



Translational Kinetic Energy of a Particle

Tuesday, October 5, 2021 1:17 PM

- For an ideal gas:

$$KE_{avg} = \frac{1}{2} m v_{rms}^2 = \frac{1}{2} m \frac{3rT}{M} = \frac{3}{2} K_B T$$

- At a given temperature T, all ideal gas molecules have the same linear KE

Mean Free Path

Tuesday, October 5, 2021 1:23 PM

- Given an ideal gas, the mean free path λ is the average distance traveled by the molecules

$$\lambda = \frac{\text{length of path}}{\text{number of collisions}} \approx \frac{v\Delta t}{\pi d^2 v \Delta t \frac{N}{V}}$$

where d is the diameter of the particles
N is the number of particles
V is the Volume

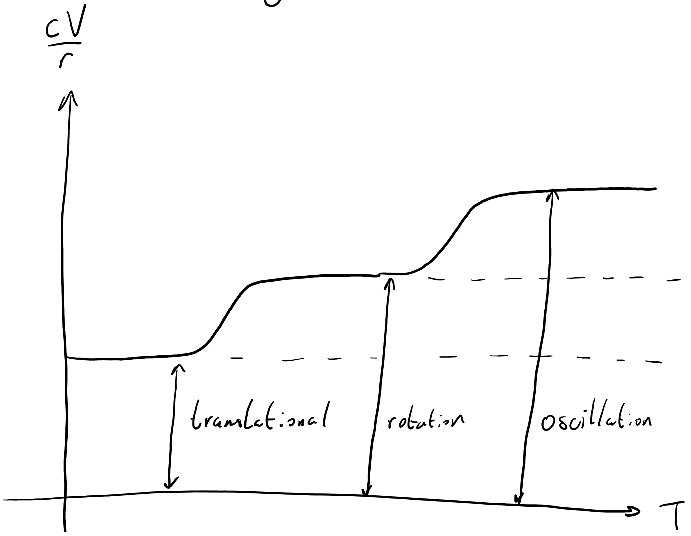
$$= \frac{1}{\sqrt{2}\,\pi d^2 \frac{N}{V}}$$

Modes

Tuesday, October 5, 2021 1:34 PM

- Every different, independent place energy can be transfered is called a mode or degree of freedom
- Monatomic molecule (He) is like a point, and has 3 modes
- Diatomic molecule (O_2) has 3 modes + 2 rotational = 5 modes

For most gasses:



Atoms, Degrees of Freedom, Cv, Cp, gamma

Thursday, October 7, 2021 1:03 PM

# atoms	DOF	C_v	C_p	$\gamma = \frac{C_v}{C_p}$
1	3	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{3}$
2	5	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{7}{5}$
3+	6	3	4	$\frac{4}{3}$

Total KE of Monotomic Ideal Gas

Tuesday, October 5, 2021 1:39 PM

- Average kinetic energy of a gas:

  $KE_{avg} = \frac{1}{2} kT$ where k is boltzmann constant

- Given a monotomic ideal gas:

  $E_{int} = \frac{3}{2} nrT$

- Energy per mode:

  $\frac{E}{mode} = \frac{1}{2} nrT$

Molar Specific Heat, Eint/Q

Tuesday, October 5, 2021 1:40 PM

- Molar specific heat, C, and molar specific heat at a constant volume C_V:

- For a <u>constant volume</u> process:

$Q = nC_V \Delta T$, and since $\Delta E_{int} = Q$, then

$$nC_V\Delta T = \frac{3}{2} nr \Delta T$$

thus: $C_V = \frac{3}{2} r = 12.5 \ J/mol \cdot K$

- In general $C_V = \frac{f}{2} r$ where f is the modes of the gas:

$\Delta E_{int} = nC_V\Delta T$

Note $Q = nC_V\Delta T$ only when volume is constant because $\Delta E_{int} = Q - W$

But $\Delta E_{int} = nC_V\Delta T$ for any volume change

- For a <u>constant pressure</u> process:

$Q = nC_p\Delta T$, and since $\Delta E_{int} = Q - W$, and $W = p\Delta V = nr\Delta T$

$$\Delta E_{int} = Q - W = nC_p\Delta T - nr\Delta T = n\Delta T(C_p - r)$$

since $\Delta E_{int} = nC_V\Delta T$:

$$C_p - r = C_V \rightarrow C_p = C_V + r$$

Adiabatic Process

Thursday, October 7, 2021 12:54 PM

- In adiabatic process, Q is 0 thus:

$\Delta E_{int} = -W$, then $pV^{\gamma} = \text{constant}$ where $\gamma = \frac{C_p}{C_v}$

where constant can be calculated from an initial condition

since $p = \frac{nrT}{V}$, then $\left(\frac{nrT}{V}\right)V^{\gamma} = \text{constant} \rightarrow TV^{\gamma-1} = \text{constant}$

Entropy, Engines

Thursday, October 7, 2021 1:26 PM

- Measure of how much disorder in a system, symbol S
- Given a system, there are some number of ways to arrange the system

$$W = \frac{n!}{(m!\,(n-m)!}$$ where n is the number of particles, m are the number particles in a particular state

- Multiplicity of system configuration to entropy:

$$S = k_B \ln W$$ where W are the number of microstates

- But! Factorials are large, so we can use:

$$\Delta S = S_f - S_i = \int_i^f \frac{\partial Q}{T}$$, since $\partial Q = \partial E_{int} + \partial W$

then $$\partial S = \int \frac{\partial E_{int}}{T} + \int \frac{\partial W}{T}$$ or $$\Delta S = \frac{\Delta E_{int}}{T} + \frac{\Delta W}{T}$$

since $\Delta E_{int} = nC_V \Delta T$ and $\Delta W = P \cdot \Delta V$ and $T = \frac{PV}{nr}$

then $$\Delta S = nC_v \cdot \int \frac{\partial T}{T} + nr \int \frac{\partial V}{V} = nC_v \ln\left(\frac{T_f}{T_i}\right) + nr \ln\left(\frac{V_f}{V_i}\right)$$

- For Isothermal processes:

$$\Delta S = \frac{Q}{T}$$

Second Law of Thermodynamics

Tuesday, October 12, 2021 12:55 PM

- Entropy of an isolated system never decreases. Entropy either increases until the system reaches equilibrium or remains the same if the system is in equilibrium.
- When two systems at different temperatures interact, heat always flows from the hottest to the coldest
- For any closed system: $\Delta S \geq 0$

Heat Engines

Tuesday, October 12, 2021 1:08 PM

Def A heat engine is a closed cycle device that extracts heat from a hot reservoir, does useful work, exhausts heat to cold reservoir.

- Closed cycle so it periodically returns to initial state (state variables return to the initial values)

- Performance of the engine ε:

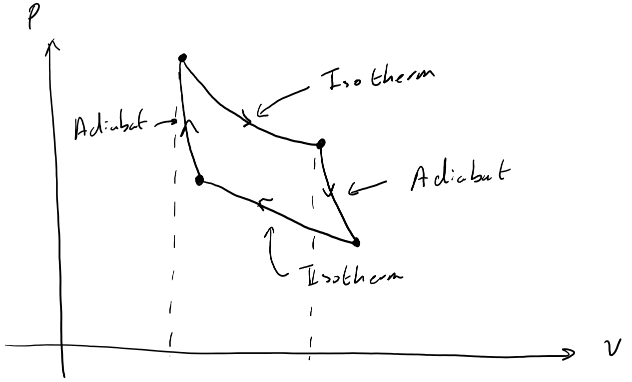
$$\varepsilon = \frac{W}{Q_{in}} = \frac{\text{Work Output}}{\text{Heat Input}}$$

, since process is cyclical: $\Delta E_{int} = 0$, $Q_H - Q_o = W$

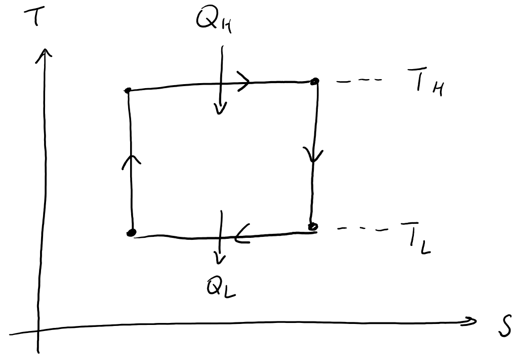
Carnot Cycles

Tuesday, October 12, 2021 1:33 PM

- To increase efficiency of an engine, must have reversible process
  - frictionless mechanical interaction with no heat transfer $(Q = 0)$
  - thermal interactions are isothermal processes $\Delta E_{int} = 0$
- Any engine that uses these two processes are Carnot Engines
- Carnot engine is a perfectly reversible engine, maximum possible thermal efficiency



→



$$\varepsilon = \frac{\text{Area of cycle}}{\text{Area under Isotherm}}$$

$$\Delta E_{int} = Q - W, \quad W = Q, \quad W = |Q_H| - |Q_L|$$

$$\Delta S = \Delta S_H + \Delta S_L = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L} = 0$$

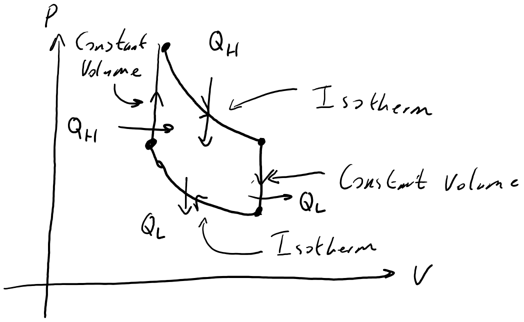
$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

$$\varepsilon = \frac{W}{Q_H} = \frac{|Q_H| - |Q_L|}{Q_H} \qquad \varepsilon_c = 1 - \frac{T_L}{T_H}$$

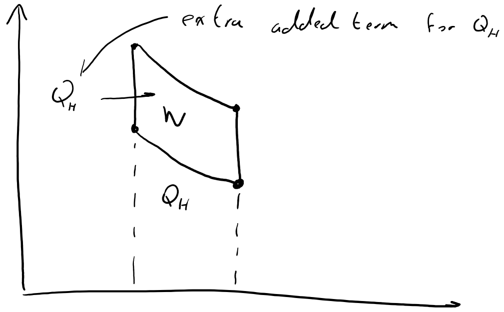
Stirling Cycles

Thursday, October 14, 2021 1:05 PM

- Similar to carnot cycle, but less efficiency



$$\varepsilon = \frac{W}{Q_H}$$

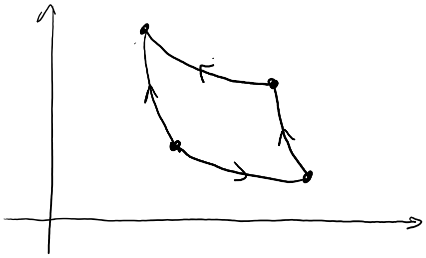


Constant volume: $Q = nC_V \Delta T$

Refrigerators

Thursday, October 14, 2021 1:09 PM

- Refrigerators transfer heat from cold object to hot object, consumes W to do so
- Opposite direction process as carnot or stirling engine:



$$\text{Efficiency } K = \frac{Q_L}{W} = \frac{\text{Heat input}}{\text{Work consumed}} = \frac{Q_L}{|Q_H| - |Q_L|}$$

$$K_{carnot} = \frac{T_L}{T_H - T_L}$$

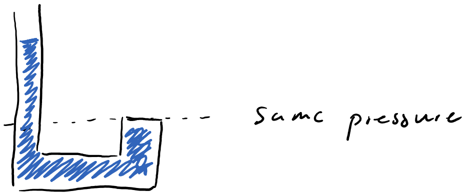
Fluidynamics

Tuesday, October 19, 2021 12:47 PM

Pressure of Fluids

Tuesday, October 19, 2021 12:42 PM

- $P = \rho g h$ where ρ = density, h = hieght relative to gravitational pull
- Pressure does not depend on the shape of the container
- Two types of pressure: Guage and Absolute
  - absolute: $P = P_{guage} + 1 \text{ atm}$
  - guage : pressure relative to reference point
- Pressure at the same height of connected fluid are the same



- Pressure of fluid as Force:

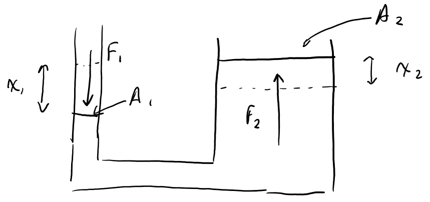
$$P = \frac{\Delta F}{\Delta A} = \frac{\partial F}{\partial A} \quad \text{thus} \quad \int p\,\partial A = \int \frac{\partial F}{\partial A}\,\partial A \rightarrow \int p\,\partial A = F$$

Pascal's Principle

Tuesday, October 19, 2021 1:15 PM

- Liquids are incompressible
- Pascal's Principle: a change in pressure to a fluid will be applied to the fluid everywhere

Ex:



$P_1 = P_2$

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

and

$W_1 = W_2$, $P_1 \cdot x_1 = P_2 \cdot x_2$

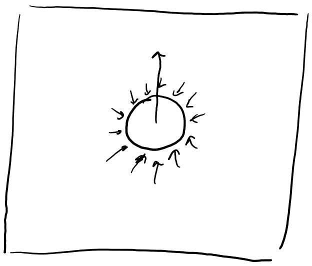
$\frac{F_1}{A_1} x_1 = \frac{F_2}{A_2} x_2$

where $\frac{F_1}{F_2}$ = mechanical advantage

Archimede's Principle, Bouyancy

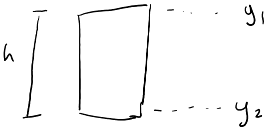
Thursday, October 14, 2021 1:29 PM

- When an object is submerged in fluid it will be given an upward force:



less pressure at top than bottom, so net upward force

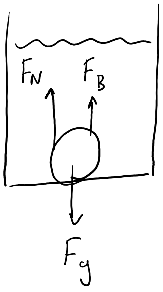
- B, bouyancy force: $B = \rho g V$ where ρ = fluid density, V= Volume of submerged object



$$\Delta P = P_2 - P_1 = \rho g (y_2 - y_1)$$

$$\frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}}$$

- If object in fluid touching a surface:



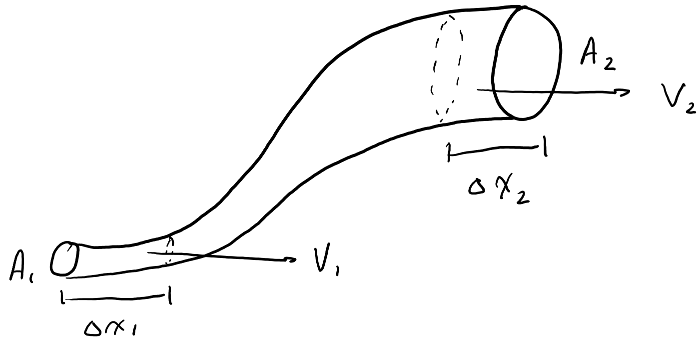
$F_B + F_N = F_g$ where F_N is the apparent weight

Continuity Equation, Fluid Flow Through Pipe

Thursday, October 21, 2021 1:11 PM

- An Ideal Fluid is incompressable:

  if a fluid flows through a pipe:



$$A_1 \cdot V_1 = A_2 \cdot V_2$$

$$P_{guage} = \frac{1}{2}\rho\left(v_1^{\,2} - v_2^{\,2}\right)$$

- Laminar flow → equal velocity at any point

  Turbulent flow → unequal smoothness at any point

Bernoulli's Principle, Equation

Tuesday, October 19, 2021 1:34 PM

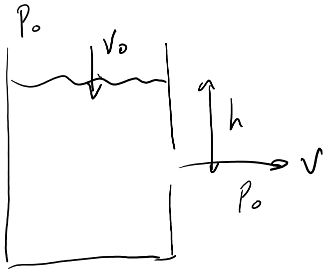
- Recall:

$$P = \frac{F}{A} = \frac{F \cdot \Delta x}{A \cdot \Delta x} = \frac{\text{Work}}{\text{Volume}} = \frac{W}{V}$$

- Thus: Pressure is energy density

and $\frac{E_{total}}{V} = P + \frac{1}{2}\rho v^2 + \rho g h$ = constant where P is pressure at a specific point
ρ is density
h is depth
v is velocity of flow

- For a tank with water with a hole:



since $P_o + \frac{1}{2}\rho V_o^2 + \rho g h = P_o + \frac{1}{2}\rho V^2 + \rho g(0)$

P_o = 1atm, we assume $V_o = 0$

$$2\rho g h = \rho v^2$$

$$V = \sqrt{2gh}$$

Waves

Tuesday, October 26, 2021 1:27 PM

- In general any wave can be:

  $A\cos(\omega t + \phi)$ where A is the amplitude, ω is the angular frequency, ϕ is the phase

- Generally, each point on a wave does not move laterally, just up/

- Carries energy and momentum from one spatial location to another

- Requires a medium to travel through

<u>Def</u> frequency of wave $= \frac{\text{completed cycle}}{\text{second}} = \frac{2\pi}{\text{period}} = 2\pi f$ where f is the frequency

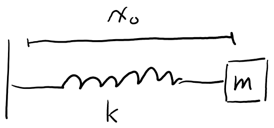
<u>Types</u> Tranverse: displacement perpendicular to travel - amplitude is height of displacement

Longitudinal: displacement in direction of travel - amplitude is maximum compression

Mass on Spring

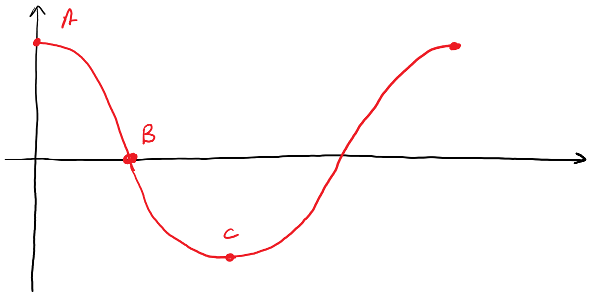
Tuesday, October 26, 2021 12:33 PM

- Given a spring with a mass is attached:



then $x = x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$ harmonic oscillator

where $\sqrt{\frac{k}{m}} = \omega$ the angular frequency



at point A : $PE = max$
$KE = 0$

at point B : $PE = 0$
$KE = max$

at point C : $PE = max$
$KE = max$

$KE = \frac{1}{2} m v^2$ $PE = \frac{1}{2} k x^2$ $KE + PE = E_{total}$, which must be constant

velocity at point x : $\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k x_{max}^2 = \frac{1}{2} k v_{max}^2 \rightarrow \underline{v^2 = \frac{k}{m}\left(x_{max}^2 - x^2\right)}$

Particle Waves, Wave Velocity

Thursday, October 28, 2021 12:29 PM

Def The equation of a particle wave is:

$y(x,t) = A \cdot \sin(kx + \omega t + \phi)$ where $k = \frac{2\pi}{\lambda}$ where λ is the wavelength and ϕ is the phase

remembering that $\omega = 2\pi f$

then the velocity of the wave is $V = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$

Note V is always dependent on the medium, so $\lambda \cdot f$ is always constant so when frequency changes, then the wavelength changes to compensate

Note the phase difference between two points is: $\Delta\phi = 2\pi \frac{\Delta x}{\lambda}$

Note Angular frequency: ω is in $\frac{rad}{s}$, and linear frequency: f is in Hz

$\omega = \frac{2\pi}{T} = 2\pi f$

String Waves

Tuesday, November 2, 2021 12:44 PM

Def to change the speed of a wave on a string, change the tension or change the density:

$V_{string} = \sqrt{\frac{F_T}{u}}$ where F_T is the tension force, u is the mass per unit length of the string

Def the equation for the wave is the same as SHM wave.

thus: $y(x, t) = y_{max} \sin(kx + \omega t + \phi)$ where k, ω are SHM and $y_{max} = ?$

String Boundary Conditions, String Standing Waves

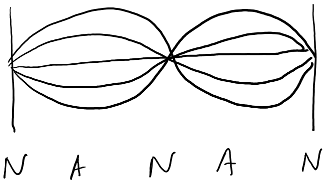
Tuesday, November 2, 2021 1:27 PM

Def when a wave hits a boundary, some or all of the wave is reflected

if the end is fixed, the amplitude is inverted:

if the end is not fixed, the amplitude is not inverted:

Def when waves are reflected back, the waves can interfere. at proper frequencies, the waves will create standing waves:



N = Nodes : places where the wave has no displacement

A = Anti-Nodes : places where the wave has maximum displacement

For a string when $L = \frac{k}{2}\lambda$ for integer k, then the standing wave has k anti-nodes

thus $\lambda = \frac{2}{k}L$ for a specific standing wave frequency

since $v = \lambda f$, then $f = \frac{v}{\lambda} = \frac{kv}{2L}$ for string with length L

the amplitude of both waves is $\frac{1}{2}A$ where A is the amplitude of the standing wave

Sound Waves

Thursday, November 4, 2021 12:44 PM

Def Sound Waves are longitudinal waves, so the density of particles changes:

Def the displacement of a particle along the wave is:

$s(x,t) = s_m \cos(kx + \omega t + \phi)$ where k, ω are SHM and $s_m = ?$

and the pressure at any point in the wave is phase shifted:

$p(x,t) = p_m \sin(kx + \omega t + \phi)$ where k, ω are SHM and $p_m = I$ (intensity)

Def the speed of sound in some medium: $V = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus constant

generally, speed of sound is solid > liquid > gas

in some gas with speed of sound at Standard Temp/Pressure: $V = V_{STP} \cdot \sqrt{\frac{T}{273K}}$

in air $V_{STP} = 343$ m/s

Def the amplitude of sound is the intensity $I = \frac{1}{A} \cdot \frac{\Delta E}{\Delta t} = \frac{P}{A}$ with units $\frac{W}{m^2}$ and is always perpendicular to the surface of the wave fronts

Def I scales with distance. For example, if $A = 4\pi r^2$, then $I = \frac{P}{4\pi r^2}$

Def we can measure sounds in dB and intensity level $\beta = (10\,dB)\log\left(\frac{I}{I_0}\right)$

where I_0 is threshold of hearing $= 1 \times 10^{-12}$

Doppler Effect

Thursday, November 4, 2021 1:44 PM

Def the apparent effect of a sound wave changing frequency because of a moving source

Idea Remembering $f = \frac{V}{\lambda}$, then if the source of the wave is moving, then

$$f = \frac{V_{wave} \pm V_{source}}{\lambda} = f_{source} \cdot \left(\frac{V_{wave}}{V_{wave} \mp V_{source}} \right)$$

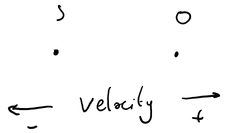
use (-) when source is moving toward, (+) when moving away

Def If the source and observer are both moving:

$$f_o = f_s \left(\frac{V_{sound} \pm V_o}{V_{sound} \mp V_s} \right)$$

where <u>top sign means toward</u>, <u>bottom sign means away</u>

Alternatively:



for both V_o and V_s

Sound/Air Standing Waves

Tuesday, November 9, 2021 1:30 PM

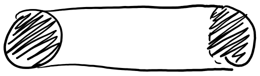
Def Standing waves can be created in a tube of air.

Each end can be open or closed, so 4 cases total

If an end is closed, then there must be a node.

If an end is open, then there must be an anti-node.

Cases: Closed-Closed:



$f_n = \frac{nv}{2L}$ $L = \frac{n}{2}\lambda$ for $n = 1, 2, 3, 4\ldots$

Open-Closed:

$f_n = \frac{nv}{4L}$ $L = \frac{n}{4}\lambda$ for $n = 1, 3, 5, 7\ldots$

Open-Open:

$f_n = \frac{nv}{2L}$ $L = \frac{n}{2}\lambda$ for $n = 1, 2, 3, 4\ldots$

Superposition, Interference

Tuesday, November 2, 2021 1:23 PM

Def If two waves meet and pass through each other they obey superposition: we add together their individual displacements

Idea If two waves are in phase with same frequency, then they add together and become stronger by constructive interference

If two waves are out of phase with same frequency, then they add together and cancel each other out by destructive interference

Def Cause of interference is the path length difference $\Delta r = r_2 - r_1$

if $\Delta r = n\lambda$ where n is an integer, then the waves will be in phase

if $\Delta r = (n + \frac{1}{2})\lambda$ where n is an integer, then the waves will be out of phase

generally, the phase difference: $\phi = 2\pi \frac{\Delta r}{\lambda}$

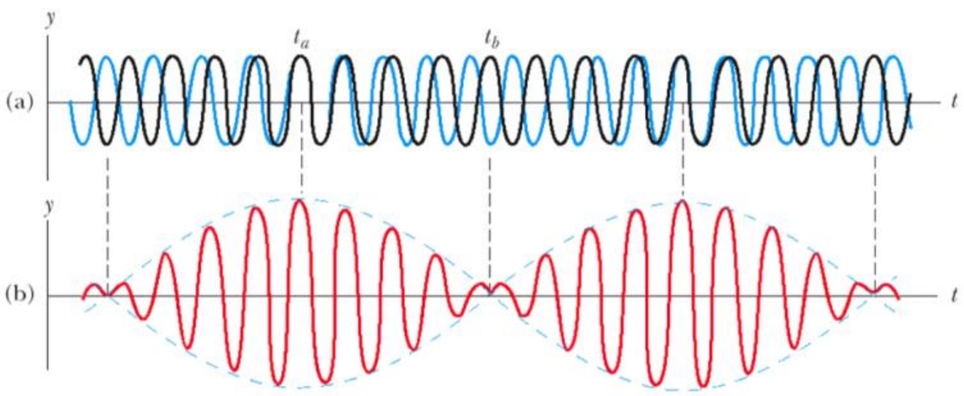
The amplitude of the resulting wave created by two identical waves traveling in the same direction:

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

Beats

Tuesday, November 16, 2021 12:38 PM

Idea If we overlay two waves of similar frequency:



Def If two waves have frequencies f_1, f_2 then the beats have frequency $|f_1 - f_2|$

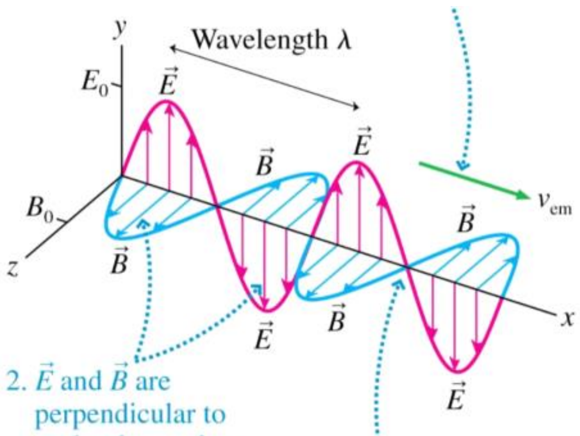
Electromagnetic Waves

Tuesday, November 16, 2021 12:47 PM

Def Electromagnetic waves are two waves: electric and magnetic

They are tranverse waves perpendicular to each other

The direction the wave propagates: $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$



2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.

3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

Def Equations for a light wave:

$$\vec{E} = E_y \hat{j} = E_{max} \sin(kx - \omega t + \phi)\hat{j}$$

$$\vec{B} = B_z \hat{k} = B_{max} \sin(kx - \omega t + \phi)\hat{k}$$

as always: $k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T} = 2\pi f$

And: $c = \frac{E_{max}}{B_{max}}$

Def EM waves always travel at the same speed c:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.0 \times 10^8 \text{ m/s}$$

Idea Light is both a particle and a wave. Particle-Wave duality.

Energy of Light, Spectrum

Tuesday, November 16, 2021 1:04 PM

Def The energy of a photon is: $E = hf$ where $h = 6.63 \times 10^{-34}$

and is shared by the electric and magnetic waves equally.

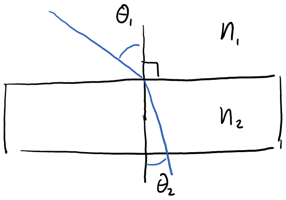
Def Recall: $v = c = \lambda f$, thus as f increases, λ decreases. This creates a spectrum of EM waves

Def The intensity of any wave: $I = \frac{\text{Power}}{\text{Area}} = |\vec{S}| = \left| \frac{\vec{E} \times \vec{B}}{\mu_0} \right|$

Refraction, Snell's Law

Tuesday, November 16, 2021 1:25 PM

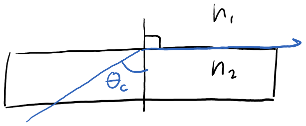
Def When light crosses from one medium to another :



then $n_1 \sin \theta_1 = n_2 \sin \theta_2$

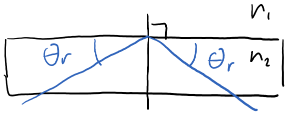
where n_1, n_2 are the index of refraction

Def When Θ_1 reaches a critical angle, the light is totally internally refracted:

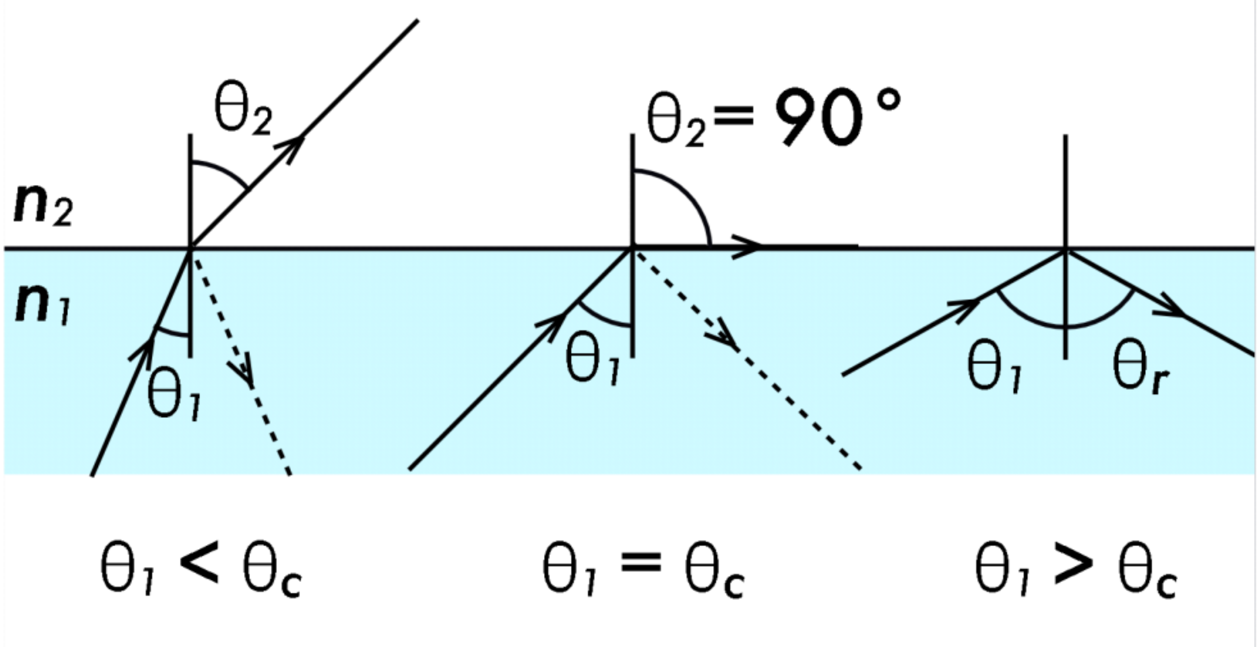


where $\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$ and only occurs when $\frac{n_1}{n_2} < 1$

and where the refracted ray is the same as the incident ray



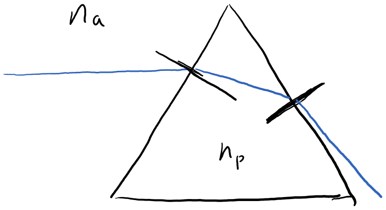
Note



Prisms, Index of Refraction

Thursday, November 18, 2021 12:52 PM

Idea Given an equilateral prism with refractive index n_p:



Just apply snell's law twice.

But, we expect a white light to spread and create the color spectrum.

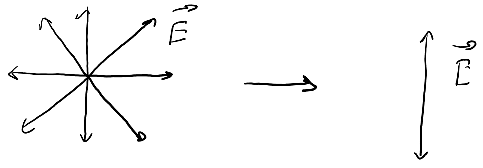
Def The index of refraction depends on the wavelength of the light.

Polarization, Brewster's Angle

Thursday, November 18, 2021 1:09 PM

<u>Def</u> Most light will contain multiple distributions of $\vec{E}$ and $\vec{B}$. Each atom produces a wave with its own electric field.

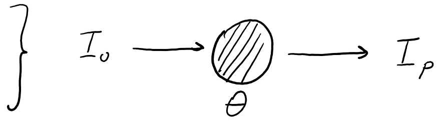
<u>Def</u> We can polarize light by passing through a polarizer:



and we can observe a polarized wave with an analyzer.

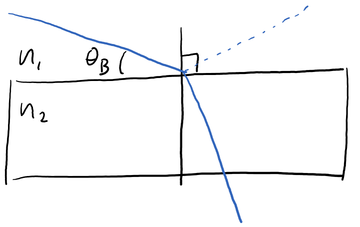
<u>Def</u> Given some unpolarized wave: $I_p = \frac{1}{2} I_0$

<u>Def</u> Given some polarized wave: $I_p = I_0 \cos^2 \theta$



<u>Note</u> Light can be polarized by electrons in atoms. This is why the sky looks blue.

<u>Def</u> Given two mediums and some polarized light: The Brewster's angle



where $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

then the polarization of the reflected ray is perpendicular to the incident ray.

Optical Systems

Thursday, November 18, 2021 1:39 PM

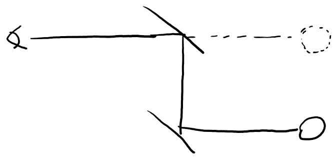
Idea Light rays can be diverted by optical systems.

Def An optical system can be made from mirrors and lenses.

Def A real image is where the light rays actually pass through the image points

A virtual image is where the light rays do not pass through the image points

real image



virtual image

Def Given some optical system component and an image:

The object distance p is the distance from object to component

The image distance i is the distance from image to component

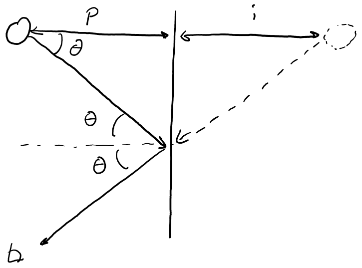
The lateral magnification m is the ratio of image size to object size

To find where an image is formed, it is always necessary to follow at least two rays of light.

Mirrors (Straight, Spherical)

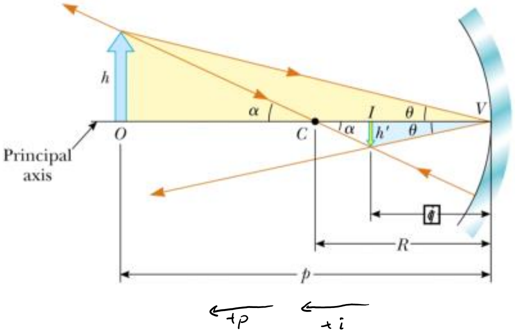
Tuesday, November 23, 2021 12:52 PM

Def Given the straight mirror:



then: $p = -i$, $h = 1$

Def Given the concave spherical mirror:

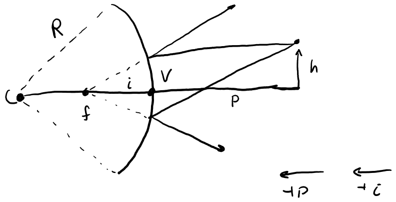


where C is the center, R is the radius, f is the focal point $= \frac{1}{2}R$

where all distances are relative to V

then: $\frac{1}{p} + \frac{1}{i} = \frac{2}{R} = \frac{1}{f}$ and $m = -\frac{i}{p}$

Def Given the convex spherical mirror:



where C is the center, R is the radius, f is the focal point $= -\frac{1}{2}R$

where all distances are relative to V

then: $\frac{1}{p} + \frac{1}{i} = \frac{2}{R} = \frac{1}{f}$ and $m = -\frac{i}{p}$

Summary:

Sign Conventions for Mirrors

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	p	+	−		
Image location	$\boxed{i}$	+	−		
Focal Length	f	+ ↑ Concave	− ↑ Convex		
Image height	h'			+	−
Magnification	M			+	−

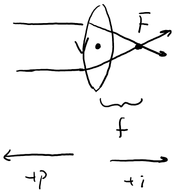
where: $\frac{1}{p} + \frac{1}{i} = \frac{2}{R} = \frac{1}{f}$

and $M = \frac{h'}{h} = -\frac{i}{p}$

Lenses (Converging, Diverging)

Tuesday, November 23, 2021 1:40 PM

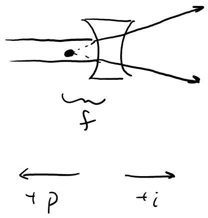
Def Given a converging lense:



where f is the focal length = positive

then $\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$ and $m = -\frac{i}{p}$

Def Given a diverging lense:



where f is the focal length = negative

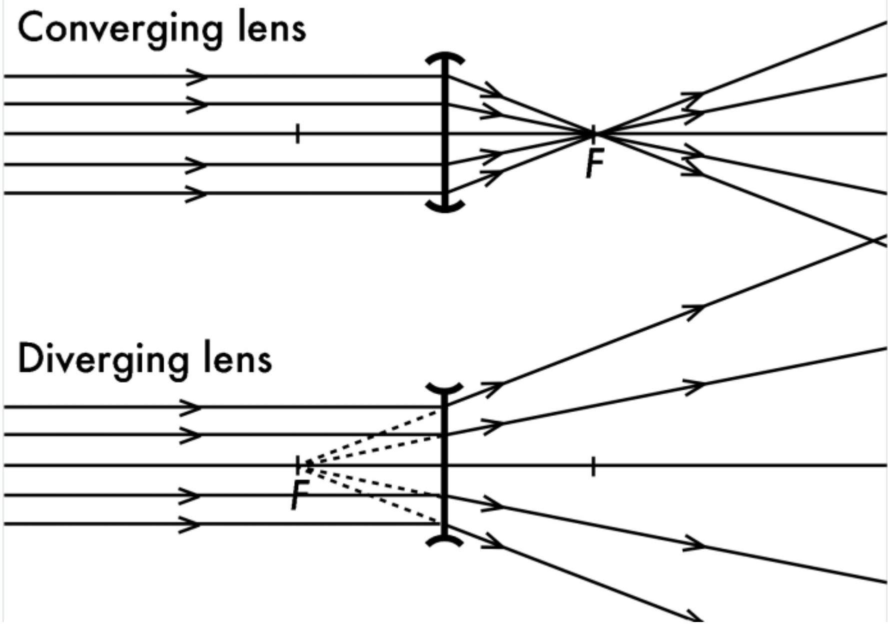
then $\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$ and $m = -\frac{i}{p}$

Summary

$$\boxed{\frac{1}{p} + \frac{1}{i} = \frac{1}{f}}$$

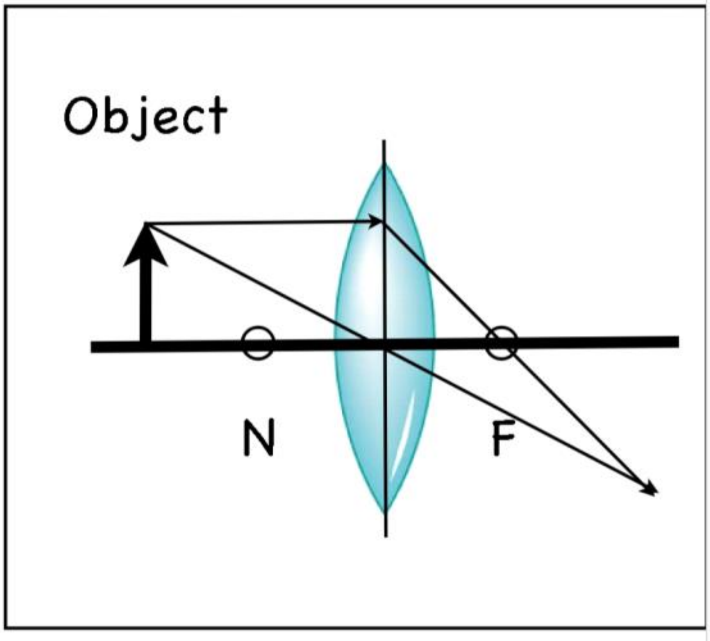
f { +: converging lens
-: diverging lens

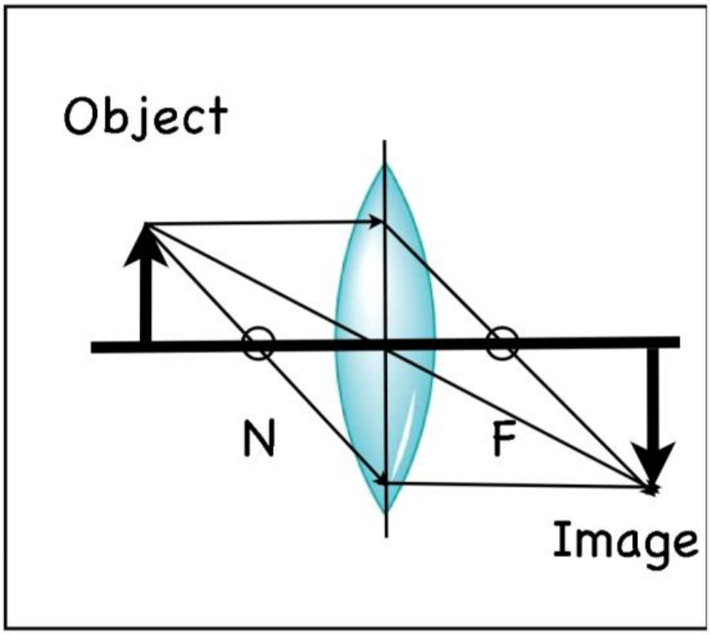
p, i { +: real object/image
-: virtual object/image



Ray Diagrams For Lenses

Tuesday, November 30, 2021 12:35 PM





Combination of Lenses

Tuesday, November 30, 2021 1:14 PM

When two lenses are placed next to each other the light rays from the object will enter one lens then the other.

The image produced by the first lens is calculated as though the second lens is not present.

The light then approaches the second lens as if it had come from the image of the first lens.

The image of the first lens is treated as the object of the second lens!!!!!

The image formed by the second lens is the final image of the system.

Def The total magnification: $M_{tot} = \Sigma M_n$

Interference of EM Waves

Tuesday, November 30, 2021 1:45 PM

Def In order to have sustained interference, there must be coherence, which is a constant relationship of phase between sources.

Def Two in phase sources exhibit constructive interference

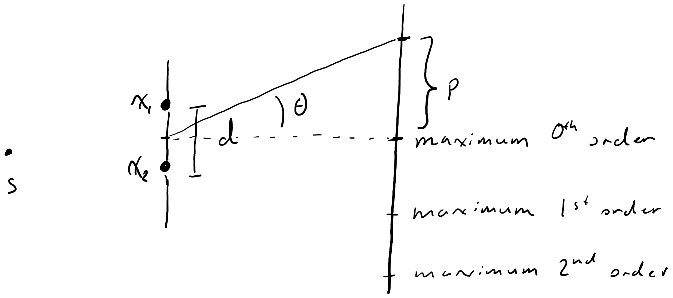
Two out phase sources exhibit destructive interference

Def $\Delta l = n\lambda$: if $n = k$ then constructive, if $n = k + \frac{1}{2}$ then destructive

Double Slit Experiment

Thursday, December 2, 2021 12:33 PM

Def Given an EM source:



$\Delta = d\sin\theta$

where Δ is the path length distance

when $d\sin\theta = m\lambda$ then constructive

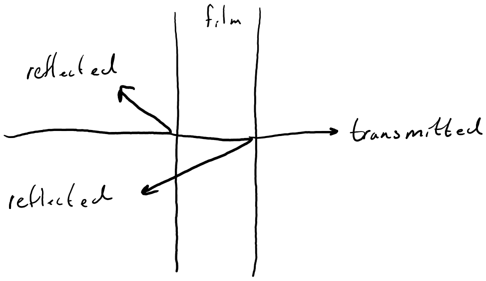
when $d\sin\theta = (m+\frac{1}{2})\lambda$ then destructive

Each slit will create their own light source when not directly observed

Interference in Thin Films

Thursday, December 2, 2021 12:49 PM

Def When white light passes through a thin film, it is interfered:



one wave will undergo a 180° phase shift when going from low to high index of refraction.

Thus: given a thin film of thickness t:

$2nt = (m + \frac{1}{2})\lambda$ where m, n are integers, then constructive interference

$2nt = m\lambda$ then destructive interference

Thin Film Interference Problem Solving strategy:

1) Identify the thin film causing the interference.

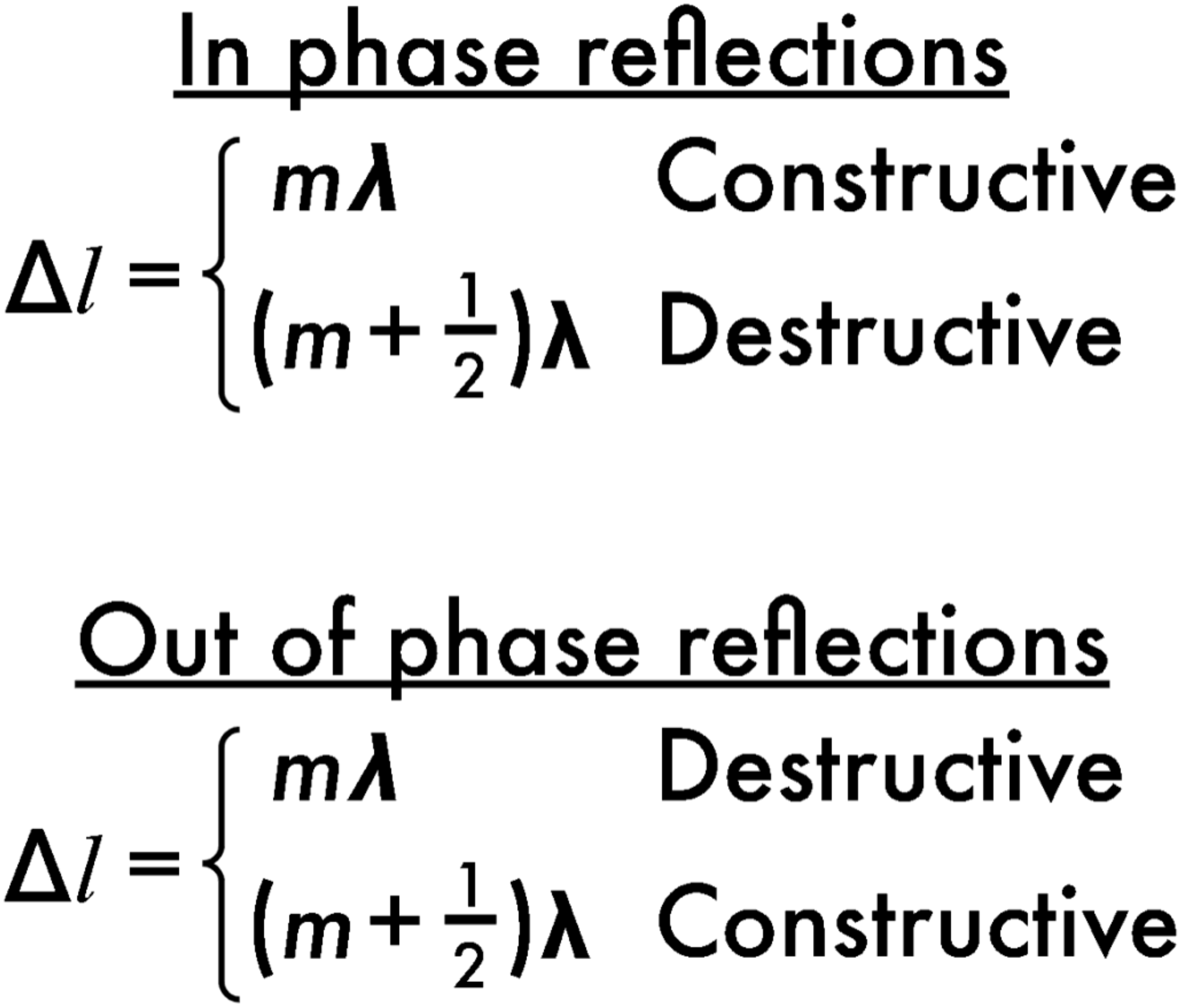
2) Determine the indices of refraction in the film and the media on either side of it.

3) Determine the number of phase reversals: zero, one or two.

4) If the interference is constructive with 0 or 2 phase reversal then use a path length difference of integral multiples of λ (use odd half multiple of λ for 1 phase reversal).

In-Phase/Out-Phase Interference

Thursday, December 2, 2021 1:05 PM



In phase reflections

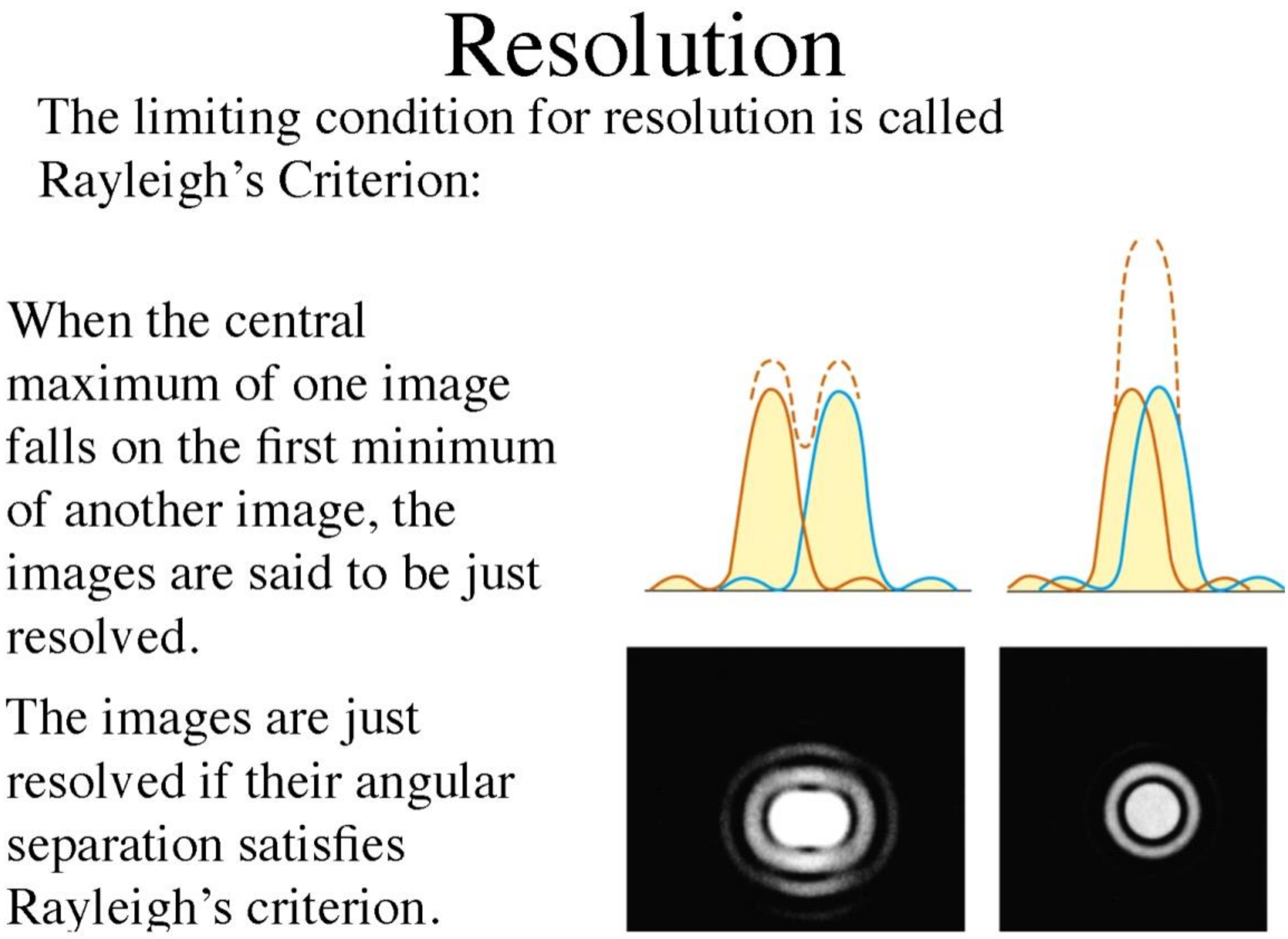
$$\Delta l = \begin{cases} m\lambda & \text{Constructive} \\ (m + \frac{1}{2})\lambda & \text{Destructive} \end{cases}$$

Out of phase reflections

$$\Delta l = \begin{cases} m\lambda & \text{Destructive} \\ (m + \frac{1}{2})\lambda & \text{Constructive} \end{cases}$$

Resolution

Thursday, December 2, 2021 1:13 PM



Resolution

The limiting condition for resolution is called Rayleigh's Criterion:

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

The images are just resolved if their angular separation satisfies Rayleigh's criterion.

X Ray Scattering

Thursday, December 2, 2021 1:15 PM

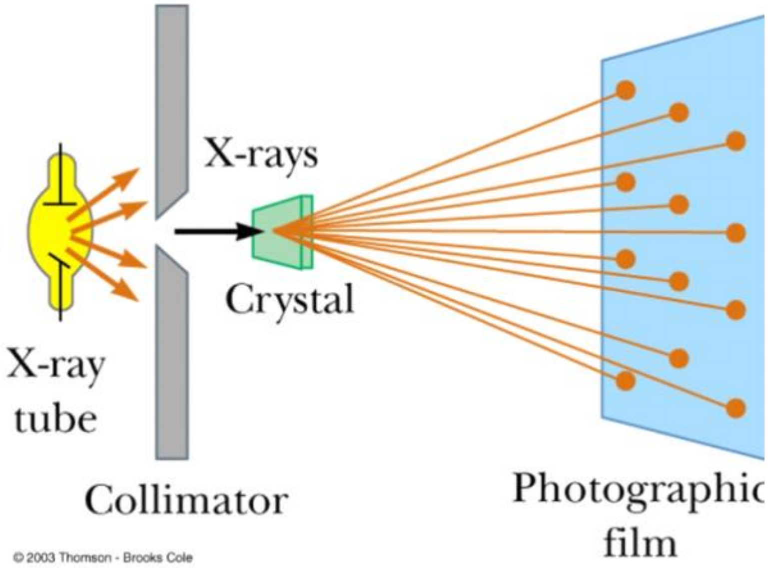
X-Ray Scattering

If you shoot a beam of X-rays at a crystal onto a photographic film, the diffracted radiation will have sections of high intensity.

These sections correspond to constructive interference.

This array of spots is called a von Laue pattern.

Since X-rays are just a form of light, these spots are caused by a path length difference.



© 2003 Thomson - Brooks Cole